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Might Dark Matter and Energy be Intrinsic Properties of Space?

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Abstract It is shown that if a volume element V, of space is assumed to have intrinsic energy E, then basic principles of mechanics, thermodynamics and special relativity lead to the equation of state: $E = pV$, where p is the pressure. When this equation of state is incorporated in the Einstein equations it leads to the prediction that the orbital speed of matter circling a visible galaxy embedded in a spherical galactic halo should be relativistic, in disagreement with observations. However, it also leads directly to the interesting notion that the inertial mass of such a medium can be understood as a resistance to being compressed via Lorentz contraction. It is then shown that the mathematical structure of thermodynamics suggests another plausible definition of pressure if we allow for the possibility that the intrinsic energy of spacetime may not be described by the same work-energy relationship as ordinary matter. This additional possibility leads to the equation of state: $E = -pV$. While both of these equations of state describe forms of energy that are quite unlike ordinary energy, neither alone is able to account for observed rotational velocity curves of matter orbiting visible galaxies. Therefore, the possibility that space has two distinct components of energy is investigated. This results in a plausible, two-component equation of state in which the former equation of state is tentatively identified as the "dark matter" (DM) component, the latter as the "dark energy" (DE) component. The effective equation of state of space, accounting for the presence of both components, may then be written in the form: $p = w\varepsilon$, where ε is the total energy density, p the total pressure, and w represents the fractional excess of DM over DE (and therefore satisfies: $-1 \le w \le +1$). Given the wide range of possible spacetime properties implied by this equation it appears to be a viable candidate for explaining observations presently attributed to the presence of both DM and DE. Specifically, the static, spherically symmetric solution of Einstein's field equations, neglecting effects of ordinary matter, predicts the inverse r^2 distribution of intrinsic space energy required to explain observed constant rotational velocity curves for matter in circular orbits around visible galaxies embedded within spherically symmetric galactic halos. The proposed equation of state is also capable of describing regions of space undergoing accelerated expansion as regions where DE is dominant (i.e., $w < -1/3$).

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1 Introduction

Although Einstein's general relativity (GR) has withstood the experimental and observational tests of nearly a century, we should not lose sight of the fact that it remains incomplete as a fundamental classical field theory. I refer here not to the fact that it is not a quantum field theory, but to the fact that the stress-energy tensors commonly employed in GR are essentially phenomenological representations of ordinary bulk matter. If the left side of Einstein's equations describes metric properties of spacetime, then, from a conceptual consistency standpoint, the right side of the equations should also describe (or at least include) the stress-energy tensor of spacetime. Apparently, Einstein tacitly assumed that the intrinsic stress-energy tensor of "empty" spacetime per se is zero. While this may have been the simplest reasonable assumption in accord with the observational evidence available at that time, an abundance of evidence presently compels us to postulate the existence of "dark matter" (DM) and "dark energy" (DE), together comprising about 96% of the energy of the universe, in order to retain our faith in the validity of GR [1, 2]. But no known forms of matter or energy seem to be viable candidates for either DM or DE, and neither observation nor experiment provides us with any direct (non-gravitational) evidence of their existence. In other words, it seems entirely consistent with all observational evidence to suppose that the observed "anomalies" in galactic motions, as well as the acceleration of the cosmic expansion, are attributable to an as yet unspecified intrinsic stress-energy tensor of "empty" space (i.e., the vacuum). The primary aim of this paper is to determine the simplest form of the equation of state of such intrinsic space energy, consistent with well established principles of elementary physics, which could explain the observed intra- and inter-galactic motions of matter and the acceleration of the cosmic expansion which are presently attributed to the influences of DM and DE respectively. The effects of the approximately 4% of cosmic energy comprised of ordinary matter (i.e., baryons, leptons and photons) will be disregarded. Several authors have considered black holes and naked singularities as constituents of dark matter. Scalar fields have been considered to explain dark energy problem. For example, see [3–11] and references cited therein.

2 Derivation of an Equation of State from Basic Principles of Conventional Physics

Let's begin by explicitly avoiding the assumption that space is "empty" (i.e., has zero vacuum energy). Rather, let us suppose a spatial volume element, V_0, has a nonzero intrinsic energy, E_0. Now consider an observer, initially at rest relative to the volume element under consideration, who begins to accelerate past it. We know from relativity theory that, relative to the observer, our volume element will contract and its energy will increase as the observer's speed increases. More precisely, if an element of space having volume V_0 and energy E_0 relative to an observer at rest is observed from a reference frame that is moving with relative speed v, its instantaneous volume relative to such an observer will be:

$$V = [1 - (v^2/c^2)]^{1/2} V_0, \tag{1}$$

and its instantaneous energy will be

$$E = [1 - (v^2/c^2)]^{-1/2} E_0. \tag{2}$$

Since the pressure is not assumed to be zero, basic principles of mechanics state that the work done by the pressure during a small, adiabatic contraction of the volume is related to the corresponding change in energy of the element by:

$$dE = -pdV. \tag{3}$$

That is, if the standard work-energy relation of mechanics is to remain valid for an accelerating observer, the gain in energy of our volume element must be accounted for by the work done on it locally by the pressure of the surrounding space.

While the work-kinetic energy theorem of Newtonian particle mechanics permits us to relate work done by net external forces on a point mass in a vacuum (presumed to be at zero pressure) to changes in its "energy of motion" (i.e., kinetic energy), a new conceptual paradigm is necessitated in relativistic continuum mechanics by the fact that the pressure may not generally be assumed zero, and so must be considered to do work on an element of the medium undergoing volume change due to Lorentz contraction. The pressure of our volume element must therefore be calculated as the negative derivative of the total energy of the element with respect to the volume of the element in order for the standard work-energy relation to remain a locally valid, Lorentz invariant expression [12].

In order for the energy and volume of our element to change relative to an accelerating observer in accordance with (1), (2), and (3), the pressure must be

$$p = -\frac{dE}{dV} = -\frac{dE/dv}{dV/dv}. \tag{4}$$

Or, upon taking the derivatives,

$$p = -\frac{E_0}{V_0[1-(v^2/c^2)]} = \frac{E}{V}. \tag{5}$$

We find that this leads to the equation of state:[1]

$$p = \varepsilon, \tag{6}$$

where ε is the intrinsic energy density of space. Let us examine the nature of the postulated intrinsic energy of space as described by (6). The energy described by this equation of state is quite unlike any form of ordinary matter or energy observed in the laboratory. For example, in the equation of state for an ultra relativistic ideal gas, the pressure approaches $1/3$ of the energy density as the mean thermal energy of the particles becomes much greater than their rest energy [15]. We see that the intrinsic energy described by (6) has three times the pressure, for a given energy density, of an ultra relativistic ideal gas. It is also apparent that this form of energy has the greatest possible proportionality coefficient, w, in an equation of state of the form: $p = w\varepsilon$, where ε is the energy per unit volume. The speed of a pressure wave in a relativistic medium is equal to c times the square root of the derivative of the pressure with respect to the energy density [16]. Since this derivative is unity for the equation of state (6), the speed of a pressure wave in such a medium is equal to the speed of light. Therefore no greater value of w would be consistent with basic principles of relativity and causality, since that would result in a signal speed greater than c.

[1] This equation of state is sometimes referred to as the "stiff" equation of state. It was first proposed by Zeldovich as a limiting case to describe nuclear matter in its most dense possible state [13]. Also, see [14].

An intensive measure of a medium's resistance (per unit volume) to being compressed is its bulk modulus, $B \equiv \rho(dp/d\rho)_s$ where ρ is the mass density, and the derivative is taken at constant entropy. Using Einstein's mass-energy equation in the form $\varepsilon = \rho c^2$, we find that $B = \rho c^2 = p$. Therefore, the compressibility, defined as the inverse of the bulk modulus, becomes zero in the non-relativistic limit $c \to \infty$. The intrinsic energy field described by equation of state (6) is therefore the relativistic generalization of a non-relativistic, incompressible medium.

3 An Aside on the Nature and Origin of Inertia of a Mass Element of "Stiff" Matter

The paradigm shift employed to derive equation of state (6) also provides us with an interesting new conceptual perspective on the nature of inertial mass. Just as work must be done on ordinary matter to compress it, work would have to be done on the mass of an element of space to force it to undergo Lorentz contraction when accelerated. (Try to imagine accelerating an entire galaxy, including the dark matter (i.e., intrinsic space energy) halo in which it is embedded.) As is readily seen from the previously derived relationships, the mass of an element of space described by (6) is given by: $m = BV/c^2$. Thus, the inertial mass of an element of space energy is directly proportional to its bulk modulus, which is its resistance (per unit volume) to being compressed via Lorentz contraction during acceleration. Similarly, during deceleration, inertial mass may be thought of as the resistance to being expanded via Lorentz expansion. To the best of the author's knowledge, this is an entirely new perspective on the nature of inertial mass.

4 Can Dark Matter be "Stiff"?

We are now in a position to make a preliminary assessment as to whether the postulated intrinsic space energy described by (6) is a viable candidate to explain some of the observed large scale features of our universe. Since we have already alluded to the dark matter (DM) halos believed to engulf most galaxies as possible manifestations of intrinsic space energy, let's see if there are solutions to the equations of GR, with the intrinsic energy of space described by (6), which could account for the observed motions of matter in orbit around some galaxies.

Some of the earliest and most impressive observations to suggest the existence of dark matter are those regarding galaxy rotation curves. Such curves show the velocity of matter rotating about galactic centers as a function of distance from the centers. At large distances, well beyond most of the visible matter of a galaxy, it would be expected, based on Newton's laws (and GR), that the rotational velocity would fall off with radial distance as the square root of the inverse of r. However, observations of many galaxies show no such decrease with r. In fact, the orbital velocities often remain nearly constant with distance [17]. This evidence has lead astronomers to postulate that such galaxies are embedded in large, spherically symmetric halos of unseen "dark" matter (DM). In order to account for such constant velocity rotation curves using Newtonian mechanics, the amount of DM mass enclosed within a sphere of radius r would have to increase linearly with r, implying that the mass density of DM would have to fall off inversely as r^2. Since the observations imply that the mass of the visible matter of such galaxies is typically a small fraction ($\sim 10\%$) of the implied total mass of their halos we should be able to approximate the observed constant velocity rotation curves far from galactic centers by neglecting the presence of the visible matter core and assuming only the presence of a spherically symmetric distribution of the hypothesized energy of space, in hydrostatic equilibrium, and satisfying (6).

Fortunately, the well-established equations of structure for spherical, relativistic stars are directly applicable to this situation. The relevant equations (in geometrized units) are [18]:

$$\begin{aligned}
\frac{dm(r)}{dr} &= 4\pi r^2 \rho(r), \\
-\frac{dp(r)}{dr} &= [\rho(r) + p(r)]\left(\frac{m(r) + 4\pi r^3 p(r)}{r^2(1 - 2m(r)/r)}\right), \\
\frac{1}{2}\frac{dv(r)}{dr} &= -\frac{1}{\rho(r) + p(r)}\frac{dp(r)}{dr} = \frac{m(r) + 4\pi r^3 p(r)}{r^2(1 - 2m(r)/r)}.
\end{aligned} \tag{7}$$

The functions $m(r)$ and $v(r)$ determine the geometry inside the halo as given by the following spherically symmetric, time-independent metric in spherical coordinates:

$$ds^2 = -e^v(r)dt^2 + c^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{8}$$

with

$$e^{-\lambda(r)} = 1 - \frac{2m(r)}{r}. \tag{9}$$

For the equation of state $p = \rho$ $(= \varepsilon$ here) we find, by direct substitution that (7) have as solutions (still in geometrized units):

$$\begin{aligned}
p(r) = \rho(r) &= \frac{1}{16\pi r^2}, \\
m(r) &= \frac{r}{4} + C_1, \\
v(r) &= 2\log[2r] + C_2.
\end{aligned} \tag{10}$$

When expressed in SI units, the solution for the mass density is:

$$\rho(r) = \frac{c^2}{16\pi G r^2} = \frac{2.68 \times 10^{25}\ \mathrm{Nm^2/kg^2}}{r^2}. \tag{11}$$

Although this solution has the r dependence required to produce constant velocity galaxy rotation curves, the uniqueness of the numerical constant predicts a single universal value of the velocity constant for all such curves. But the observational evidence does not indicate that objects in orbit around galaxies exhibiting constant velocity rotational curves all have the same velocity constant. Furthermore, if we assume this velocity to be non-relativistic and estimate its value by equating the (Newtonian) gravitational attraction to the centripetal force for objects in circular orbit about the mass distribution given by (11), we get:

$$v = \sqrt{4\pi G(2.68 \times 10^{25}\ \mathrm{Nm^2/kg^2})} = 1.5 \times 10^8\ \mathrm{m/s} = c/2. \tag{12}$$

Clearly, our assumption that this orbital velocity is not relativistic is unjustified. But observed orbital velocities are in fact non-relativistic. For example, the velocity constant for the nearby Andromeda galaxy (M31), is approximately 200,000 m/s [18]. We are forced to conclude that an intrinsic energy of space having equation of state (6) cannot adequately explain the observed constant velocity galactic rotation curves.

5 Derivation of an Equation of State of Space from a Plausible Alternative Work-Energy Relation

At this point, we could simply concede that our original hypothesis that space has an intrinsic energy is inconsistent with the observational evidence. However, there is an implicit assumption in the reasoning leading to (6), namely that the thermodynamic relation (3), which is valid for ordinary matter, is also valid for the intrinsic energy of space. When referring to the measurable pressure of ordinary bulk matter, it is generally understood that we are referring to a statistically averaged value of the force per unit area exerted on a surface (real or imagined) that is impenetrable to impacts by the elementary constituents (e.g., molecules) of such matter. The pressure of ordinary matter is, therefore, not defined locally, at each point in spacetime as a continuous classical field should be, but is a statistical property, like the temperature, that is properly applied only to large numbers of particles over times that are much longer than the mean time between impacts with the impenetrable surface used for measurement. Also, because particle impacts deliver impulses only in the direction of their motion, absolute pressure can be positive only, the zero of pressure corresponding to no impacts at all. If we are to allow for the possibility that the intrinsic energy of space may be quite unlike the energy of ordinary matter, we should not assume (as we have implicitly in (4)) that the same relationship between energy, volume and pressure that holds for ordinary matter must also hold for the postulated intrinsic energy of space. In fact, the underlying mathematical structure of thermodynamics suggests a very different relationship. To see what this relationship is, we must examine the postulational formulation of thermodynamics. For the convenience of the reader, the postulational formulation presented in reference [19] is briefly summarized in the following paragraphs.

The fundamental postulate of thermodynamics (as applied to ordinary matter) is that: "There exist particular states (called equilibrium states) of simple systems that, macroscopically, are characterized completely by the internal energy U, the volume V, and the mole numbers $N_1, N_2, \ldots, N_r$ of the chemical components" [19]. However, in the present case, we are not considering ordinary matter, composed of moles of chemical components, but the intrinsic energy of space itself. We therefore restate the fundamental postulate of thermodynamics (as applied to the intrinsic energy of space) as follows: "There exist particular states (called equilibrium states) of space energy that, macroscopically, are characterized completely by the total energy E and the volume V." Strictly speaking, this approach limits us to static, equilibrium states of space, but we proceed under the assumption that vast regions of space are changing sufficiently slowly under sufficiently weak gravitational fields that they are adequately described by such states.

The second basic postulate of standard thermodynamics is that: "There exists a function (called the entropy S) of the extensive parameters of any composite system, defined for all equilibrium states and having the following property. The values assumed by the extensive parameters in the absence of an internal constraint are those that maximize the entropy over the manifold of constrained equilibrium states" [19].

The third postulate states that: "The entropy of a composite system is additive over the constituent subsystems. The entropy is continuous and differentiable and is a monotonically increasing function of the energy" [19]. And the forth, and final, postulate states that: "The entropy of any system vanishes in the state for which $(\frac{\partial E}{\partial S})_V = 0$ (that is, at the zero of temperature)" [19].

Mathematically, the first three postulates permit us to write the total space energy, E, as a function of the remaining extensive parameters:

$$E = E(S, V). \tag{13}$$

We refer to (13) as the fundamental equation in the energy representation. The first differential of the total energy is therefore:

$$dE = \left(\frac{\partial E}{\partial S}\right)_V dS + \left(\frac{\partial E}{\partial V}\right)_S dV. \tag{14}$$

The coefficients of the differentials on the right side of (14) are the intensive parameters of standard thermodynamics. The coefficient of the entropy differential is, by definition, the temperature, T. The third postulate requires that the temperature thus defined is nonnegative and the forth postulate tells us that the entropy is zero when the temperature is zero. We assume the temperature of the quiescent vacuum to be zero and therefore are left with:

$$dE = \left(\frac{\partial E}{\partial V}\right)_{S=0} dV. \tag{15}$$

At this point, mathematical symmetry suggests that the coefficient of the volume differential in (15) should be definable as an important intensive thermodynamic parameter, namely the pressure, and indeed it is. Although our experience with ordinary matter forces us to define the coefficient of the volume differential in (15) as the negative pressure, when investigating an entirely new type of energy we are free to follow our sense of mathematical aesthetics and postulate that the coefficient of the volume differential in (15) represents the pressure of the hypothesized intrinsic energy of space. Of course, this is at variance with our usual understanding of the work-energy relation for ordinary matter, but the validity of this choice for the description of the relationship between the hypothesized energy and pressure of space can be judged only by the degree to which it leads to predictions that are consistent with observation. Instead of (4), (5) and (6), we now have:

$$p \equiv +\frac{dE}{dV} = \frac{dE/dv}{dV/dv}. \tag{16}$$

Upon taking the derivatives we have,

$$p = \frac{-E_0}{V_0[1-(v^2/c^2)]} = -\frac{E}{V}. \tag{17}$$

In stark contrast to (6) we now have the equation of state:

$$p = -\varepsilon. \tag{18}$$

If we seek spherically symmetric solutions to (7) using (18), we find that there are none. We are therefore forced to conclude that an intrinsic energy of space having equation of state (18) cannot adequately explain the observed constant velocity galactic rotation curves.

Although (18) is not adequate to describe the hypothesized intrinsic energy of space, it too describes a form of energy that is quite unlike any form of bulk matter or energy observed in the laboratory. In order for this form of energy to be positive, its pressure must be negative. This would result in the stress-energy tensor having a negative trace. Therefore, it describes a form of energy which tends to repel itself gravitationally and expand. In this respect it is similar in effect to the cosmological constant, leading to spatial expansion [20]. It is not, however, required to have constant density. Observations have of course informed us that there are indeed vast regions of accelerating spatial expansion in our universe [1, 2].

6 Might Space Have Two Distinct Components of Intrinsic Energy?

Although neither (6) nor (18) alone appears adequate to describe an intrinsic energy of space that also can account for the observational anomalies presently attributed to the presence of dark matter and dark energy, the two types of energy that they describe do, when taken together, have properties that are, qualitatively at least, similar to those presently attributed to DM and DE. Like the energy described by (6), DM seems to pull itself together gravitationally and become more concentrated in certain regions of space, just as ordinary matter does. Observations also suggest that DM tends to form, and remain in, spherical concentrations, even when it surrounds spiral and other disk-shaped galaxies. This suggests that it is not subject to the same dissipation mechanisms as ordinary matter, again in accord with our hypothesis that it is a field property of spacetime itself rather than of a particulate nature. The DE, on the other hand, appears to exist primarily in vast intergalactic regions undergoing continuous, accelerating expansion, behavior which could be explained, at least qualitatively, by an intrinsic energy described by (18). Could it be that both (6) and (18) describe the intrinsic energy of space?

The derivations of (6) and (18) rely only on Lorentz invariance and the two simplest possible thermodynamic definitions of pressure, one of which, (6), is fully confirmed by our experience with ordinary matter. Apparently, we must hypothesize the existence of both of these forms of energy if we are to retain hope of explaining both DM and DE as due to the intrinsic energy of space. In accord with previous remarks, let us tentatively identify the energy described by (6) as DM energy, and that described by (18) as DE. We then assume that both DM and DE are components of the intrinsic energy of space. In other words, we assume that the total intrinsic energy of space is the sum of these two components, so that we have for the total energy density

$$\varepsilon = \varepsilon_{\mathrm{DE}} + \varepsilon_{\mathrm{DM}} \tag{19}$$

and, for the total pressure

$$p = p_{\mathrm{DE}} + p_{\mathrm{DM}}. \tag{20}$$

Using (6) and (18), (20) becomes

$$p = \varepsilon_{\mathrm{DM}} - \varepsilon_{\mathrm{DE}}. \tag{21}$$

The postulated equation of state of the intrinsic energy of space may therefore be written in the form

$$p = w\varepsilon, \tag{22}$$

where

$$w \equiv \frac{\varepsilon_{\mathrm{DM}} - \varepsilon_{\mathrm{DE}}}{\varepsilon_{\mathrm{DM}} + \varepsilon_{\mathrm{DE}}}. \tag{23}$$

We see that the value of w indicates the fractional imbalance between DM energy and DE and may range from -1 to $+1$, depending on the extent to which DE or DM dominates. From the perspective of cosmology, this wide range of possible values for w is more than adequate to accommodate the construction of reasonable models. In regions of space where $w < -1/3$, the trace of the stress-energy tensor would be negative, and it would be expected, in accordance with GR, that the stress-energy of spacetime would be effectively gravitationally repulsive and expand. Therefore our equation of state (22) is of a sufficiently general form to describe the observed expansion of space in the large intergalactic voids.

On the other hand, for $w > -1/3$ the trace of the stress-energy tensor would be positive, and it would be gravitationally attractive and tend to become concentrated and spherically symmetric in certain regions (assuming that both $\varepsilon_{\mathrm{DM}}$ and $\varepsilon_{\mathrm{DE}}$ are positive). Therefore (22) might also explain the formation of DM halos if it can be shown that regions in which $w > -1/3$ would be expected to occur during the early evolution of the cosmos.

To put our hypothesis to the test, let us try once again to explain the constant velocity rotation curves observed in the vicinity of many galaxies. To do this we again look for solutions to (7), but this time we use (22) and (23). We again seek inverse r^2 solutions for ε, and therefore set both $\varepsilon_{\mathrm{DM}}$ and $\varepsilon_{\mathrm{DE}}$ proportional to one over r^2. Specifically, if we denote the DM and DE components of the assumed inverse r^2 energy density solutions to (7) by

$$\varepsilon_{\mathrm{DM}} = \frac{C_{\mathrm{DM}}}{r^2} \quad \text{and} \quad \varepsilon_{\mathrm{DE}} = \frac{C_{\mathrm{DE}}}{r^2}, \tag{24}$$

where C_{DM} and C_{DE} are constants. We find from (23) that

$$w \equiv \frac{C_{\mathrm{DM}} - C_{\mathrm{DE}}}{C_{\mathrm{DM}} + C_{\mathrm{DE}}}. \tag{25}$$

In this case, the radial dependence of w disappears, and it becomes a constant. For a linear equation of state in the form $p = w\rho$, where w is constant, we find, using Mathematica, that (7) have as solutions (in geometrized units):

$$p(r) = \frac{w^2}{2\pi r^2(1 + 6w + w^2)}, \tag{26}$$

$$\rho(r) = \frac{w}{2\pi r^2(1 + 6w + w^2)}, \tag{27}$$

$$m(r) = \frac{2wr}{1 + 6w + w^2} + C_1 \tag{28}$$

and

$$v(r) = \frac{4w \log[(1 + w)r]}{1 + w} + C_2. \tag{29}$$

We notice that when the energy density of DM and DE are equal (i.e., $w \to 0$) both the total pressure and energy density of spacetime vanish, and the flat spacetime of Minkowski satisfies the equations of GR. On the other hand, we see that the intrinsic energy of space satisfying our equation of state (22) can indeed be distributed in a stable, spherically symmetric way so as to satisfy the equations of GR, while at the same time having solutions whereby the density falls off inversely as r^2, thus resulting in constant orbital velocities, independent of r, in accord with many observations. Furthermore, there are an infinite number of such solutions, corresponding to an infinite number of possible values of w, and therefore an infinite number of possible constant values for orbital velocity. To estimate the value of w for, say, Andromeda (M31), which we know has a non-relativistic rotational velocity constant, we assume that $p(r) \ll \rho(r)$, or, equivalently, from (22), that $w \ll 1$. Going back to SI units, we find that the mass density solution in (27) becomes:

$$\rho(r) \cong \frac{c^2 w}{16\pi G r^2} = \frac{(2.68 \times 10^{25}\ \mathrm{Nm^2/kg^2})w}{r^2}. \tag{30}$$

Now we can obtain an approximate relationship between w and measured values of galaxy rotational velocity constants by equating the centripetal force to the (Newtonian) gravitational attraction for an object in circular orbit around the mass distribution (30):

$$w \cong 4\frac{v^2}{c^2}. \tag{31}$$

Using the measured value: $v = 2 \times 10^5$ m/s, we obtain an estimate of $w \cong 1.8 \times 10^{-6}$ for the fractional excess of DM energy over DE in the halo of Andromeda. Since the rotational velocity curve for M31 is quite typical of spiral galaxies (including the Milky Way), the estimated fractional excess of DM energy over DE would be estimated to be similarly minuscule in many other spiral galaxy halos.

If we look far from all observed distributions of ordinary matter, to the vast intergalactic voids, we find that these spaces are mostly undergoing expansion, and that, on average (over the entire cosmos), the rate of expansion is accelerating [2]. As previously noted, the hypothesized intrinsic energy of space can describe such regions within the framework of GR only if $w < -1/3$ in (22), in which case the trace of the stress-energy tensor is negative. Thus, if there were localized regions present in the early universe where

$$w \equiv \frac{\varepsilon_{\mathrm{DM}} - \varepsilon_{\mathrm{DE}}}{\varepsilon_{\mathrm{DM}} + \varepsilon_{\mathrm{DE}}} < -1/3 \tag{32}$$

or, equivalently, where $\varepsilon_{\mathrm{DE}} > 2\varepsilon_{\mathrm{DM}}$, then we would expect them to have undergone considerable expansion by now, to be expanding presently, and to continue expanding indefinitely, comprising an ever-increasing fraction of the universe. The energy associated with galaxies (including halos) would therefore ultimately become relatively negligible on a cosmic scale, and our universe would be expected to attain a composition satisfying (32). This would be a universe comprised of at least two thirds DE. In as much as present observations indicate that our universe is comprised of about 70% DE and is undergoing accelerated expansion, we have apparently already entered this stage of cosmic evolution.

7 Summary

The primary aim of this investigation has been to determine the simplest possible equation of state of a hypothetical intrinsic energy of space that could plausibly describe the DM and DE of the universe as such. The preliminary assessments presented above suggest that this aim has been achieved. It is the author's hope that others will continue this line of investigation, subjecting these results to further scrutiny and comparison with observations. It is also my hope that this new line of inquiry will lead to a deeper understanding of the nature and origin of inertial mass and vacuum energy.

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